Optimization on Spheres: Models and Proximal Algorithms with Computational Performance Comparisons

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Abstract

We present a unified treatment of the abstract problem of finding the best approximation between a cone and spheres in the image of affine transformations. Prominent instances of this problem are phase retrieval and source localization. The common geometry binding these problems permits a generic application of algorithmic ideas and abstract convergence results for nonconvex optimization. We organize variational models for this problem into three different classes and derive the main algorithmic approaches within these classes (13 in all). We identify the central ideas underlying these methods and provide thorough numerical benchmarks comparing their performance on synthetic and laboratory data. The software and data of our experiments are all publicly accessible. We also introduce one new algorithm, a cyclic relaxed Douglas-Rachford algorithm, which outperforms all other algorithms by every measure: speed, stability and accuracy. The analysis of this algorithm remains open.