Nonlinear Forward-Backward Splitting with Projection Correction

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Abstract

We propose and analyze a versatile and general algorithm called nonlinear forward-backward splitting (NOFOB). The algorithm consists of two steps; first an evaluation of the nonlinear forward-backward map followed by a relaxed projection onto the separating hyperplane it constructs. The key novelty of the method is the nonlinear forward-backward step, where the backward part is based on a novel nonlinear resolvent construction. It allows for the kernel in the resolvent to be a nonlinear single-valued maximal monotone operator. This generalizes the standard resolvent as well as the Bregman resolvent, whose resolvent kernels are gradients of convex functions.

This general construction opens up for a new understanding of many existing operator splitting methods and paves the way for devising new algorithms. We show, e.g., that forward-backward-forward splitting (FBF) and forward-backward-half-forward splitting (FBHF) are special cases that rely on the nonlinearity in the nonlinear resolvent and use smaller relaxations in the projections than allowed in NOFOB. We propose long-step versions and show that synchronous projective splitting is long-step FBF applied to a specific primal-dual formulation. Time permitting, we also present a novel four operator splitting based on NOFOB and a long-step NOFOB variation that collects many separating hyperplanes before relaxed projection.